ACSC/STAT 4703, Actuarial Models II
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Homework Sheet 6

Model Solutions

Basic Questions

1. *An insurance company starts a new line of insurance in 2017, and collects a total of \$1,400,000 in premiums that year, and the estimated incurred losses for accident year 2017 are \$982,000. The premium payments are uniformly distributed over the year. An actuary is using this data to estimate rates for premium year 2020. Claims are subject to 3% inflation per year. By what percentage should premiums increase from 2017 in order to achieve a loss ratio of 0.7.*

   Since the premium payments are uniformly distributed over the year, the average proportion of a premium which is earned in 2017 is 0.5, so the earned premiums are \$700,000. The achieved loss ratio is $\frac{982000}{700000} = 1.40285714286$. To achieve a loss ratio of 0.7, the adjustment factor is $\frac{1.40285714286}{0.7} = 2.00408163266$. We also need to adjust for inflation. Since the number of in-force policies is assumed to increase linearly thoughout the year, the average discount factor is

$$\begin{aligned}
\int_0^1 2t(1.03)^t &= \int_0^1 2te^{\log(1.03)t}\,dt \\
&= \left[2t\frac{e^{\log(1.03)t}}{\log(1.03)}\right]_0^1 - \int_0^1 2\frac{e^{\log(1.03)t}}{\log(1.03)}\,dt \\
&= 2\frac{e^{\log(1.03)t}}{\log(1.03)} - 2\frac{e^{\log(1.03)}-1}{\log(1.03)^2} \\
&= \frac{2\times 1.03}{\log(1.03)} - \frac{2\times 0.03}{\log(1.03)^2} \\
&= 1.0199260311
\end{aligned}$$

   For the payments from premium year 2020, the average inflation factor from the start of 2017 is

$$\begin{aligned}
&\int_0^1 t(1.03)^{t+3} + \int_1^2 (2-t)(1.03)^{t+3}\\
=&-\frac{(1.03)^4}{\log(1.03)} + \frac{(1.03)^4-(1.03)^3}{\log(1.03)^2} + 2\left(\frac{(1.03)^5-(1.03)^4}{\log(1.03)}\right)\\
&\qquad - \left(\frac{(1.03)^5-2(1.03)^4}{\log(1.03)} + \frac{(1.03)^5-(1.03)^4}{\log(1.03)^2}\right)\\
=&1.12559076094
\end{aligned}$$

We therefore multiply the adjustment factor by $\frac{1.12559076094}{1.0199260311} = 1.10360038534$ to get an overall adjustment factor of $1.10360038534 \times 2.00408163266 = 2.21170526206$ so the premium should be increased by 121.171%.

2. *An insurer collects \$1,200,000 in earned premiums for accident year 2017. The total loss payments are \$1,052,000. Payments are subject to inflation of 4%, and policies are sold uniformly throughout the year. If the insurer's permissible loss ratio is 75%, by how much should the premium be changed for policy year 2019?*

   Assuming the number of policies in force is constant throughout the year, the average inflation factor for payments in accident year 2017 is

$$\int_0^1 (1.04)^t\,dt = \left[\frac{(1.04)^t}{\log(1.04)}\right]_0^1 = \frac{0.04}{\log(1.04)} = 1.01986926764$$

   The average inflation factor from the start of 2017 to a random loss in policy year 2019 is

$$\begin{aligned}
&\int_0^1 t(1.04)^{t+2} + \int_1^2 (2-t)(1.04)^{t+2}\\
=&-\frac{(1.04)^3}{\log(1.04)} + \frac{(1.04)^3-(1.04)^2}{\log(1.04)^2} + 2\left(\frac{(1.04)^4-(1.04)^3}{\log(1.04)}\right)\\
&\qquad - \left(\frac{(1.04)^4-2(1.04)^3}{\log(1.04)} + \frac{(1.04)^4-(1.04)^3}{\log(1.04)^2}\right)\\
=&1.12500820224
\end{aligned}$$

   The current loss ratio is $\frac{1052000}{1200000} = 0.876666666667$. The premium therefore needs to be changed by a factor $\frac{0.876666666667}{0.75} \times \frac{1.12500820224}{1.01986926764} = 1.28939034564$ or 28.94%.

3. *A workers' compensation insurer classifies companies into three sectors — manufacture, retail and services. The experience from policy year 2017 is:*

Sector	*Current differential*	*Earned premiums*	*Loss payments*
Manufacture	*2.36*	*5,230*	*2,100*
Retail	*0.91*	*4,280*	*3,900*
Services	*1*	*7,100*	*5,400*

The base premium was \$370. Claim amounts are subject to 5% annual inflation. If the expense ratio is 30%, calculate the new premiums for each sector for policy year 2020.

The observed expense ratios are $\frac{2100}{5230} = 0.401529636711$, $\frac{3900}{4280} = 0.91121495327$ and $\frac{5400}{7100} = 0.760563380282$. The new differentials are therefore $\frac{0.401529636711}{0.760563380282} \times 2.36 = 1.24593159125$ and $\frac{0.91121495327}{0.760563380282} \times 0.91 = 1.09025181724$. Adjusted to these differentials, the earned premiums for retail and manufacture are $5230 \times \frac{0.401529636711}{0.760563380282} = 2761.11111111$ and $4280 \times \frac{0.91121495327}{0.760563380282} = 5127.77777777$, making the total earned premiums 14988.8888889, so the loss ratio is $\frac{11400}{14988.8888889} = 0.760563380281$. The adjustment factor for an expense ratio of 30% is therefore $\frac{0.760563380281}{0.7} = 1.08651911469$. Since we are calculating the premiums for policy year 2020, we need to multiply by 1.05^3 to adjust for inflation. This means the new base premium is $370 \times 1.08651911469 \times 1.05^3 = \$465.38$. With the new differentials of 1.24593159125 and 1.09025181724, the premiums for Manufacture and Retail companies are $465.38 \times 1.24593159125 = \$579.83$ and $465.38 \times 1.09025181724 = \$507.38$ respectively.

Standard Questions

4. *An insurer has different premiums for male and female customers. Its experience for accident year 2017 is given below. There was a rate change on 7th April 2017, which affects some of the policies.*

Sex	*Differential before rate change*	*Current differential*	*Earned premiums*	*Loss payments*
Male	*1*	*1*	*11,200*	*9,100*
Female	*1.11*	*1.07*	*8,500*	*6,300*

Before the rate change, the base premium was \$840. The current base premium is \$960. Assuming that policies are sold uniformly over the year, calculate the new premimums for policy year 2019 assuming 5% annual inflation and a permissible loss ratio of 0.75.

7th April is the 97th day of the year (2017 is not a leap year). Assuming uniform distribution of sales, the proportion of policies in the year sold after 7th April is therefore $\frac{1}{2}\left(\frac{268}{365}\right)^2 = 0.269559016701$. We will convert all premiums to the new premium. For male, the rates are changed by a factor $\frac{960}{840} = 1.14285714286$, so we obtain the adjusted premiums by dividing the earned premiums by $0.269559016701 \times 1 + 0.730440983299 \times 1.14285714286^{-1} = 0.908694877086$. This gives the adjusted premium $\frac{11200}{0.908694877086} = 12325.3693648$. For female policyholders, the adjustment is $\frac{1.07 \times 960}{1.11 \times 840} = 1.10167310167$. Therefore we adjust the premiums by dividing by $0.269559016701 \times 1 + 0.730440983299 \times 1.10167310167^{-1} = 0.932587806449$, to get $\frac{8500}{0.932587806449} = 9114.42326526$. The adjusted loss ratios are $\frac{9100}{12325.3693648} = 0.738314587633$ and $\frac{6300}{9114.42326526} = 0.691212138898$.

The new differential is $1.07 \times \frac{0.691212138898}{0.738314587633} = 1.00173693031$. Using this differential, the adjusted premiums for female are $\frac{0.691212138898}{0.738314587633} \times 9114.42326526 = 8532.94802179$. The total adjusted premiums are 20858.3173866, so the loss ratio is $\frac{15400}{20858.3173866} = 0.738314587633$. We therefore adjust the premium by a factor $\frac{0.738314587633}{0.75} = 0.984419450177$. To adjust for inflation, the discount factor for accident year 2017 is $\int_0^1 (1.05)^{-t}\,dt = \frac{1-(1.05)^{-1}}{\log(1.05)} = 0.975996872109$, while the discount factor for policy year 2019 is

$$\begin{aligned}
&\int_0^1 t(1.05)^{-t-2} + \int_1^2 (2-t)(1.05)^{-t-2} \\
=&-\frac{(1.05)^{-3}}{\log(1.05)} + \frac{(1.05)^{-2}-(1.05)^{-3}}{\log(1.05)^2} + 2\left(\frac{(1.05)^{-3}-(1.05)^{-4}}{\log(1.05)}\right) \\
&\qquad - \left(\frac{(1.05)^{-3}-2(1.05)^{-4}}{\log(1.05)} + \frac{(1.05)^{-3}-(1.05)^{-4}}{\log(1.05)^2}\right) \\
=&0.86400897483
\end{aligned}$$

To adjust for inflation, we multiply by $\frac{0.975996872109}{0.86400897483} = 1.12961427548$. The new base premium is therefore $960 \times 0.984419450177 \times 1.12961427548 = \$1,067.53$. The new differential is 1.00173693031, so the new premium for female policyholders is $1.00173693031 \times 1067.53 = \$1,069.39$.

5. *An insurer classifies automobile insurance policyholders into male or female, and into car or motorcycle. It has the following data from policy year 2016:*

Number of policies

	car	*motorcycle*
Male	*530*	*132*
Female	*252*	*44*

loss payments

	car	*motorcycle*
Male	*\$50,400*	*\$25,800*
Female	*\$11,300*	*\$2,000*

(a) If the base classes are Male and car, the base rate is \$120, and the differentials are 0.7 for female and 1.63 for motorcycle, calculate the new premiums which give an expense ratio of 0.2 using the loss-ratio method.

Multiplying the number of policies by the premium gives the annual earned premiums

	car	motorcycle
Male	$530 \times 120 = 63,600$	$132 \times 120 \times 1.63 = 25,819.2$
Female	$252 \times 120 \times 0.7 = 21,168$	$44 \times 120 \times 1.63 \times 0.7 = 6,024.48$

The loss ratios for male and female are therefore $\frac{76200}{89419.2} = 0.852165977777$ and $\frac{13300}{27192.48} = 0.489105811607$ respectively, so the new differential for female is $0.7 \times \frac{0.489105811607}{0.852165977777} = 0.401769229297$. The loss ratios for cars and motorcycles are $\frac{61700}{84768} = 0.727869007173$ and $\frac{27800}{31843.68} = 0.873014676696$. The new differential for motorcycles is therefore $\frac{0.873014676696}{0.727869007173} \times 1.63 =$

1.95504123543. Using these differentials to balance back, with these differentials at the current base premium, we get total earned premiums of

$$120\,(530 + 132 \times 1.95504123543 + 252 \times 0.401769 + 44 \times 0.401769 \times 1.95504123543) = 110864.655529$$

and the loss ratio would be $\frac{89500}{110864.655529} = 0.807290651587$. The base premium for 2016 therefore needs to be adjusted by a factor $\frac{0.807290651587}{0.80} = 1.00911331448$. So the new base premium is $120 \times 1.00911331448 =$ \$121.09. The premium for male motorcycle drivers is $121.093597738 \times 1.95504123543 =$ \$236.74. The premium for female car drivers is $121.093597738 \times 0.401769229297 =$ \$48.65. The premium for female motorcycle drivers is $121.093597738 \times 1.95504123543 \times 0.401769229297 =$ \$95.12.

(b) Repeat part (a) based on differentials of 0.85 for female and 0.95 for motorcycle.

Multiplying the number of policies by the premium gives the annual earned premiums

	car	motorcycle
Male	$530 \times 120 = 63,600$	$132 \times 120 \times 0.95 = 15,048$
Female	$252 \times 120 \times 0.85 = 25,704$	$44 \times 120 \times 0.95 \times 0.85 = 4,263.6$

The loss ratios for male and female are therefore $\frac{76200}{78648} = 0.968873970095$ and $\frac{13300}{29967.6} = 0.443812650996$ respectively, so the new differential for female is $0.85 \times \frac{0.443812650996}{0.968873970095} = 0.389359983848$. The loss ratios for cars and motorcycles are $\frac{61700}{89304} = 0.690898503986$ and $\frac{27800}{19311.6} = 1.43954928644$. The new differential for motorcycles is therefore $\frac{1.43954928644}{0.690898503986} \times 0.95 = 1.97941059971$. Using these differentials to balance back, with these differentials at the current base premium, we get total earned premiums of

$$120\,(530 + 132 \times 1.9794106 + 252 \times 0.3893600 + 44 \times 0.3893600 \times 1.9794106) = 110,797.423125$$

and the loss ratio would be $\frac{89500}{110797.423125} = 0.807780519399$. The base premium for 2016 therefore needs to be adjusted by a factor $\frac{0.807780519399}{0.80} = 1.00972564925$. So the new base premium is $120 \times 1.00972564925 = 121.16707791$. The premium for male motorcycle drivers is $121.16707791 \times 1.97941059971 =$ \$239.84. The premium for female car drivers is $121.16707791 \times 0.389359983848 =$ \$47.18. The premium for female motorcycle drivers is $121.16707791 \times 1.97941059971 \times 0.389359983848 =$ \$93.38.