ACSC/STAT 4703, Actuarial Models II

FALL 2022
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Homework Sheet 3

Model Solutions

Basic Questions

1. *A homeowner's house is insured at \$270,000. The insurer requires 75% coverage for full insurance. The home sustains \$9,600 damage from wind. The policy has a deductible of \$4,000, which decreases linearly to zero when the total cost of the loss is \$12,000. The insurance company reimburses \$6,240. What value are they using for the houses value?*

   The deductible is $\frac{12000-9600}{8000} \times 4000 = \$1200$. Thus if the home were fully insured, the insurer would pay $9600 - 1200 = \$8,400$. Thus the home has $\frac{6240}{8400} = 0.742857142857$ coverage. This means that for full coverage a value of $\frac{270000}{0.742857142857} = \$363461.538462$ is required. Since this is 75% of the homes value, the homes value must be $\frac{363461.538462}{0.75} = \$484615.38$

2. *An insurance company has two lines of coverage in its Tennant's Insurance packages, with different expected loss ratios, and has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
Apartment	*2019*	*\$6,400,000*	*0.84*	*\$5,200,000*
	2020	*\$6,800,000*	*0.85*	*\$4,900,000*
	2021	*\$6,700,000*	*0.84*	*\$4,600,000*
House	*2019*	*\$3,500,000*	*0.77*	*\$2,100,000*
	2020	*\$4,200,000*	*0.78*	*\$1,800,000*
	2021	*\$5,300,000*	*0.76*	*\$1,900,000*

   Calculate the loss reserves at the end of 2021.

   We calculate the expected losses and the expected unpaid losses.

Policy Type	Policy Year	Expected total Losses	Losses paid to date	Reserves Needed
	2019	\$5,376,000	\$5,200,000	\$176,000
Apartment	2020	\$5,780,000	\$4,900,000	\$880,000
	2021	\$5,628,000	\$4,600,000	\$1,028,000
	2019	\$2,695,000	\$2,100,000	\$595,000
House	2020	\$3,276,000	\$1,800,000	\$1,476,000
	2021	\$4,028,000	\$1,900,000	\$2,128,000
Total				\$6,283,000

So the total loss reserves needed at the end of 2021 are \$6,283,000.

3. *The following table shows the cumulative paid losses (in thousands) on claims from one line of business of an insurance company over the past 5 years.*

Accident year	*Earned premiums*	*Development year 0*	*1*	*2*	*3*	*4*
2017	*7178*	*2294*	*3726*	*4310*	*4855*	*5232*
2018	*8589*	*2840*	*5101*	*4975*	*5691*	
2019	*6788*	*3268*	*4221*	*5198*		
2020	*8332*	*3380*	*4933*			
2021	*10094*	*3494*				

Assume that all payments on claims arising from accidents in 2016 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method

First we compute the loss development factors:

Mean

0/1 $\frac{17981}{11782} = 1.52614157189$

1/2 $\frac{14483}{13048} = 1.10997854077$

2/3 $\frac{10546}{9285} = 1.13581044696$

3/4 $\frac{5232}{4855} = 1.07765190525$

Using these values to complete the table gives the following cumulative losses:

Accident year	Development year 0	1	2	3	4
LDF		1.55237829544	1.08459657702	1.10705965765	1.05420311906
2018				5691	6133
2019			5198	5904	6362
2020		4933	5476	6219	6702
2021	3494	5332	5919	6723	7245

The future payments are the differences between consecutive years:

Accident year	Development year 0	1	2	3	4
2018				442	
2019			706	458	
2020		543	743	483	
2021	1838	587	804	522	

Average

The loss development factors are:

0/1	$\frac{1}{4}\left(\frac{3726}{2294}+\frac{5101}{2840}+\frac{4221}{3268}+\frac{4933}{3380}\right)=1.54286175591$
1/2	$\frac{1}{3}\left(\frac{4310}{3726}+\frac{4975}{5101}+\frac{5198}{4221}\right)=1.1211657155$
2/3	$\frac{1}{2}\left(\frac{4855}{4310}+\frac{5691}{4975}\right)=1.135184857$
3/4	$\frac{5232}{4855}=1.07765190525$

Using these values to complete the table gives the following cumulative losses:

Accident year	Development year 0	1	2	3	4
LDF		1.54286175591	1.1211657155	1.135184857	1.07765190525
2018				5691.000	6133
2019			5198	5900.691	6359
2020		4933	5531	6278	6766
2021	3494	5391	6044	6861	7394

The future payments are the differences between consecutive years:

Accident year	Development year 0	1	2	3	4
2018					442
2019				703	458
2020			598	747	488
2021		1897	653	817	533

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.74.

Using the mean and average LDFs from part (a), we get the following:

Development Year	Cumulative proportion of losses paid: mean LDF	average LDF	Proportion of losses paid: mean LDF	average LDF
0	0.4822878	0.4725612	0.48228775	0.47256123
1	0.7360394	0.7290966	0.25375164	0.25653542
2	0.8169879	0.8174382	0.08094854	0.08834152
3	0.9279434	0.9279434	0.11095550	0.11050526
4	1.0000000	1.0000000	0.07205657	0.07205657

This gives the following reserves for mean LDF:

Accident year	Earned premiums	Expected Total claims	Development year 0	1	2	3	4
2018	8589	6355.86					458
2019	6788	5023.12				557	362
2020	8332	6165.68			499	684	444
2021	10094	7469.56		1895	605	829	538

and the following reserves for average LDF:

Accident year	Earned premiums	Expected Total payments	Development year 0	1	2	3	4
2018	8589	6355.86					458
2019	6788	5023.12				555	362
2020	8332	6165.68			545	681	444
2021	10094	7469.56		1916	660	825	538

4. *An actuary is reviewing the following claims data:*

No. of closed claims

Acc. Year	*Development Year 0*	*1*	*2*	*3*	*4*	*Ult.*
2017	*4296*	*8282*	*9809*	*10486*	*10529*	*10792*
2018	*6067*	*9875*	*11490*	*12124*		*12449*
2019	*5636*	*9684*	*11844*			*12995*
2020	*7090*	*11637*				*14166*
2021	*9329*					*17850*

Total paid losses on closed claims (000's)

Acc. Year	*Development Year 0*	*1*	*2*	*3*	*4*
2017	*2156*	*8956*	*9879*	*10276*	*20762*
2018	*3597*	*7603*	*9046*	*17557*	
2019	*10125*	*13866*	*17338*		
2020	*10460*	*7351*			
2021	*10124*				

(a) Calculate tables of percentage of claims closed and cumulative average losses.

For percentages of claims closed, we divide the claims closed by the ultimate claims closed:

Acc. Year	Development Year 0	1	2	3	4
2017	39.8	76.7	90.9	97.2	97.6
2018	48.7	79.3	92.3	97.4	
2019	43.4	74.5	91.1		
2020	50.0	82.1			
2021	52.3				

For cumulative average losses, we just divide the second table by the first.

Acc. Year	Development Year 0	1	2	3	4
2017	502	1081	1007	980	1972
2018	593	770	787	1448	
2019	1796	1432	1464		
2020	1475	632			
2021	1085				

(b) Adjust the total loss table to use the current disposal rate.

We multiply the aggregate cumulative losses by the current disposal rate divided by the original disposal rate.

Acc. Year	Development Year 0	1	2	3	4
2017	2831	9587	9906	10300	20762
2018	3857	7874	8933	17557	
2019	12201	15285	17338		
2020	10923	7351			
2021	10124				

(c) Use the chain ladder method, with average loss development factors to estimate claim development based on the adjusted numbers. Compare this to the chain ladder method on aggregate payments on closed claims.

The average loss development factors are:

Development Year	LDF Adjusted	LDF Original
0/1	$\frac{1}{4}\left(\frac{9587}{2831}+\frac{7874}{3857}+\frac{15285}{12201}+\frac{7351}{10923}\right)=1.83841717139$	$\frac{1}{4}\left(\frac{8956}{2156}+\frac{7603}{3597}+\frac{13866}{10125}+\frac{7351}{10460}\right)=2.08498717057$
1/2	$\frac{1}{3}\left(\frac{9906}{9587}+\frac{8933}{7874}+\frac{17338}{15285}\right)=1.1006940607$	$\frac{1}{3}\left(\frac{9879}{8956}+\frac{9046}{7603}+\frac{17338}{13866}\right)=1.18108318592$
2/3	$\frac{1}{2}\left(\frac{10300}{9906}+\frac{17557}{8933}\right)=1.50259151574$	$\frac{1}{2}\left(\frac{10276}{9879}+\frac{17557}{9046}\right)=1.49052204569$
3/4	$\frac{20762}{10300}=2.01572815534$	$\frac{20762}{10276}=2.0204359673$

Using these values, we estimate the following cumulative losses:

Acc. Year	Development Year 0	1	2	3	4
2017					20762
2018				17557	35390.92
2019			17338	26051.61	52514.12
2020		7351	8091.421	12157.95	24507.66
2021	10124	18612.333	20487.038	30783.27	62052.07

Acc. Year	Development Year 0	1	2	3	4
2017					20762.00
2018				17557	35472.79
2019			17338	25842.67	52213.46
2020		7351	8682.142	12940.92	26146.31
2021		21108.41	24930.788	37159.89	75079.18

Thus the reserves are:

Acc. Year	Development Year 0	1	2	3	4
2018					17833.92
2019				8713.6073	26462.51
2020			740.4206	4066.5280	12349.71
2021		8488.333	1874.7049	10296.2281	31268.80

Acc. Year	Development Year 0	1	2	3	4
2018					17915.79
2019				8504.671	26370.79
2020			1331.142	4258.782	13205.39
2021		10984.41	3822.378	12229.101	37919.29

Standard Questions

5. *An insurance company has the following aggregate loss development data:*

Accident	*Earned*	*Development year*				
year	*premiums*	*0*	*1*	*2*	*3*	*4*
2017	*82864*	*17592*	*39598*	*57167*	*63803*	*68242*
2018	*112460*	*21110*	*47601*	*68628*	*76510*	
2019	*132278*	*25470*	*57409*	*82935*		
2020	*154944*	*31635*	*71278*			
2021	*156018*	*27332*				

From this table, it calculates the following mean loss development factors:

Development year	*LDF*
0/1	*2.253343*
1/2	*1.443419*
2/3	*1.115410*
3/4	*1.069574*

After adding a loss payment in development year 3 for loss year 2018, the reserve needed for loss year 2019, development year 3, using the chain-ladder method increases by 13094.77. How much increase in the reserve would the additional loss cause if the company were using the Bornhuetter-Fergusson method with expected loss ratio 0.79 to calculate reserves?

For the current data, the reserve needed for loss year 2019, development year 3 is $82935 \times (1.115410 - 1) = 9571.52835$. With the added loss, it is therefore $9571.52835 + 13094.77 = 22666.29835$, so the predicted cumulative payments are $82935 + 22666.29835 = 105601.29835$. Thus the new LDF for development year 2/3 is $\frac{105601.29835}{82935} = 1.27330196359$. No other LDFs are changed by the additional payment.

[We do not need to calculate the loss that was added to the 2018 Development Year 3 payments, but we can do so from the new LDF. If the added loss is x, then we have $\frac{140313+x}{125795} = 1.27330196359$, which gives $x = 19862.02051$.]

Under Bornhuetter-Fergusson, the expected total payments for accident year 2019 are $132278 \times 0.79 = 104499.62$, and the expected proportion of payments made in development year 3 for the original data is $\frac{1}{1.069574} - \frac{1}{1.069574 \times 1.115410} = 0.096738215101$. For the data with the additional payment, it is $\frac{1}{1.069574} - \frac{1}{1.069574 \times 1.27330196359} = 0.200678342779$. Therefore, the change in reserves is $104499.62(0.200678342779 - 0.096738215101) = \$10,861.70$.