# ACSC/STAT 4720, Life Contingencies II <br> Fall 2015 <br> Toby Kenney <br> Homework Sheet 5 <br> Due: Friday 20th November: 12:30 PM 

## Basic Questions

1. An insurance company sells a 5 -year annual life insurance policy to a life aged 41 , for whom the lifetable below is appropriate.

| $x$ | $l_{x}$ | $d_{x}$ |
| :---: | ---: | ---: |
| 41 | 10000.00 | 9.53 |
| 42 | 9990.47 | 11.05 |
| 43 | 9979.42 | 12.87 |
| 44 | 9966.56 | 15.05 |
| 45 | 9951.51 | 17.66 |

The annual gross premium is $\$ 385$. Initial expenses are $\$ 250$ plus $25 \%$ of the first premium. The death benefits are $\$ 230,000$. Renewal costs are $3 \%$ of each subsequent premium. The interest rate is $i=0.03$
(a) Calculate the expected net cash-flows associated with this policy (assuming no reserve). [This is the profit vector for the policy.]
(b) Calculate the expected profit margin of the policy using a risk discount rate $i=0.06$.
2. An insurance company sells a 5 -year annual life insurance policy to a life aged 56 , for whom the lifetable below is appropriate.

| $x$ | $l_{x}$ | $d_{x}$ |
| :---: | ---: | :---: |
| 56 | 10000.00 | 11.06 |
| 57 | 9988.94 | 11.98 |
| 58 | 9976.96 | 13.02 |
| 59 | 9963.94 | 14.20 |
| 60 | 9949.74 | 15.52 |

The annual gross premium is $\$ 625$. Initial expenses are $\$ 130$ plus $20 \%$ of the first premium. The death benefits are $\$ 420,000$. Renewal costs are $4 \%$ of each subsequent premium. The interest rate is $i=0.03$. Reserves are calculated on the basis $i=0.01$, with mortality following the table.
(a) Calculate the reserves.
(b) Calculate the profit signature.
(c) Calculate the Discounted payback period at a risk rate of $i=0.10$.
3. For the policy in Question 2:
(a) Calculate the reserves and profit signature for a general premium.
(b) Calculate the premium that gives an internal rate of return of $i=0.15$.
4. For the policy in Question 1, use profit testing to calculate the reserves needed to ensure that all cash flows are non-negative.

## Standard Questions

5. A couple purchase a reversionary annuity. Annual Premiums of $\$ 14,830$ are payable while both are alive. If the husband dies first, an annual life annuity of $\$ 50,000$ is payable to the wife until her death. The husband and wife are both aged 69 and both have mortality following the lifetable below. Assume both lives are independent.

| $x$ | $l_{x}$ | $d_{x}$ |
| :---: | ---: | ---: |
| 69 | 10000.00 | 99.59 |
| 70 | 9900.41 | 110.56 |
| 71 | 9789.85 | 122.59 |
| 72 | 9667.26 | 135.75 |
| 73 | 9531.51 | 150.11 |
| 74 | 9381.40 | 165.70 |
| 75 | 9215.70 | 182.57 |
| 76 | 9033.13 | 200.72 |
| 77 | 8832.41 | 220.14 |
| 78 | 8612.28 | 240.78 |
| 79 | 8371.50 | 262.54 |
| 80 | 8108.96 | 285.27 |
| 81 | 7823.69 | 308.76 |
| 82 | 7514.93 | 332.70 |
| 83 | 7182.23 | 356.71 |
| 84 | 6825.52 | 380.30 |
| 85 | 6445.22 | 402.88 |
| 86 | 6042.34 | 423.73 |
| 87 | 5618.61 | 442.05 |
| 88 | 5176.56 | 456.92 |
| 89 | 4719.64 | 467.38 |


| $r$ | $l_{x}$ | $d_{x}$ |
| :---: | ---: | ---: |
| 90 | 4252.27 | 472.44 |
| 91 | 3779.83 | 471.15 |
| 92 | 3308.68 | 462.72 |
| 93 | 2845.96 | 446.54 |
| 94 | 2399.42 | 422.39 |
| 95 | 1977.03 | 390.48 |
| 96 | 1586.55 | 351.57 |
| 97 | 1234.97 | 307.05 |
| 98 | 927.93 | 258.85 |
| 99 | 669.08 | 209.41 |
| 100 | 459.68 | 161.42 |
| 101 | 298.26 | 117.51 |
| 102 | 180.75 | 79.90 |
| 103 | 100.85 | 50.02 |
| 104 | 50.83 | 28.29 |
| 105 | 22.55 | 14.08 |
| 106 | 8.47 | 5.93 |
| 107 | 2.54 | 1.99 |
| 108 | 0.54 | 0.48 |
| 109 | 0.06 | 0.06 |

Initial expenses are $\$ 13,000$, and renewal expenses are $\$ 350$ at the start of each subsequent year while the husband is alive, and $\$ 20$ at the start of each year while the husband is dead and the wife is alive. The interest rate is $i=0.05$. Use a profit test without reserves to determine the NPV of this policy at a risk discount rate of $i=0.15$.

