ACSC/STAT 4720, Life Contingencies II
FALL 2015
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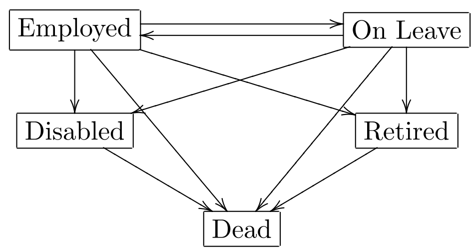
Sample Midterm Examination

This Sample examination has more questions than the actual midterm, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An insurance company is considering a new policy. The policy includes states with the following state diagram:



Which of the following sequences of transitions are possible? (Indicate which parts of the transition sequence are not possible if the sequence is not possible.)

(i) Alive—Disabled—Retired—Dead

(ii) Alive—On Leave—Retired—Dead

(iii) Alive—Retired—On leave—Dead

(iv) Alive—On leave—Alive—Retired—Dead

[5 mins.]

2. Consider a permanent disability model with transition intensities

$$\mu_x^{01} = 0.002 + 0.000005x$$
$$\mu_x^{02} = 0.001 + 0.0000004x^2$$
$$\mu_x^{12} = 0.003 + 0.000004x$$

where State 0 is healthy, State 1 is permanently disabled and State 2 is dead. Write down an expression for the probability that an individual aged 29 is alive but permanently disabled at age 56. [You do not need to evaluate the expression, but should perform basic simplifications on it.] [10 mins.]

3. A disability income model has transition intensities

$$\mu_x^{01} = 0.002$$
$$\mu_x^{10} = 0.001$$
$$\mu_x^{02} = 0.002$$
$$\mu_x^{12} = 0.004$$

State 0 is healthy, State 1 is sick and State 2 is dead. Three actuaries calculate different values for the transition probabilities and benefit values. Which one has calculated plausible values? Justify your answer by explaining what is impossible about the values calculated by the other two actuaries.

Value	Actuary I	Actuary II	Actuary III
${}_{2}p_{37}^{(00)}$	0.992036	0.992036	0.992036
${}_{2}p_{37}^{(01)}$	0.003960	0.003968	0.003964
${}_{4}p_{37}^{(01)}$	0.007857	0.007857	0.007857
${}_{4}p_{37}^{(02)}$	0.015857	0.008000	0.008000
${}_{4}p_{37}^{(12)}$	0.008000	0.015857	0.015857
${}_{2}p_{39}^{(01)}$	0.003960	0.003968	0.003964
${}_{2}p_{39}^{(11)}$	0.992054	0.992054	0.990054

[10 mins.]

4. A disability income model has the following four states:

State	Meaning
0	Healthy
1	Sick
2	Accidental Death
3	Other Death

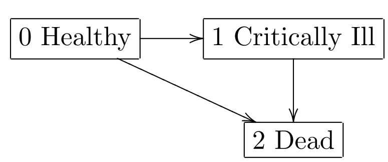
The transition intensities are:

$$\mu_x^{01} = 0.001$$
$$\mu_x^{02} = 0.002$$
$$\mu_x^{03} = 0.001$$
$$\mu_x^{10} = 0.002$$
$$\mu_x^{12} = 0.001$$
$$\mu_x^{13} = 0.003$$

You calculate that the probability that the life is healthy t years from the start of the policy is $0.2113249e^{-0.006732051t} + 0.7886751e^{-0.003267949t}$, and the probability that the life is sick t years from the start of the policy is $0.2886752e^{-0.003267949t} - 0.2886752e^{-0.006732051t}$.

Calculate the premium for a 5-year policy with premiums payable continuously while the life is in the healthy state, which pays no benefits while the life is in the sick state, but pays a benefit of \$200,000 in the event of accidental death and a benefit of \$100,000 in the event of other death. The interest rate is $\delta = 0.03$. [15 mins.]

5. Under a certain model for transition intensities in a critical illness model, with the following transition diagram:



you calculate:

$$
\begin{array}{lll}
{}_5p_{41}^{00} = 0.866102 & {}_5p_{41}^{01} = 0.0542667 & {}_5p_{41}^{02} = 0.0796309 \\
\overline{a}_{41}^{00} = 13.5501 & \overline{a}_{41}^{01} = 2.48302 & \overline{a}_{41}^{02} = 8.96688 \\
\overline{a}_{46}^{0,0} = 13.1355 & \overline{a}_{46}^{0,1} = 2.49464 & \overline{a}_{46}^{0,2} = 9.36984 \\
\overline{a}_{46}^{1,1} = 13.2984 & \overline{a}_{46}^{1,2} = 11.7016 & \\
\overline{A}_{41}^{01} = 0.196752 & \overline{A}_{41}^{02} = 0.358682 & \overline{A}_{46}^{01} = 0.202971 \\
\overline{A}_{46}^{02} = 0.374801 & \overline{A}_{46}^{12} = 0.468071 &
\end{array}
$$

where 0 is healthy, 1 is critically ill, and 2 is dead. Calculate the premium for a 5-year policy for a life aged 41, with continuous premiums payable while in the healthy state, which pays a benefit \$280,000 immediately upon death in the case of death directly from the healthy state and a benefit of \$190,000 upon entry to the critically ill state, followed by a further benefit of \$140,000 upon death after diagnosis of critical illness. Force of interest is $\delta = 0.04$. [10 mins.]

6. The following is a multiple decrement table giving probabilities of surrender (decrement 1) and death (decrement 2) for a life insurance policy:

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
49	10000.00	235.54	1.46
50	9763.00	222.44	1.55
51	9539.01	210.28	1.65
52	9327.08	198.99	1.77

A life insurance policy has a death benefit of \$400,000 payable at the end of the year of death. Premiums are payable at the beginning of each year. Calculate the premium for a 4-year policy sold to a life aged 49 if there is no-payment to policyholders who surrender their policy, and the interest rate is $i = 0.06$.

7. Update the multiple decrement table below

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
58	10000.00	176.04	2.68
59	9823.96	167.67	2.88
60	9656.29	159.84	3.10
61	9496.46	152.50	3.34
62	9343.96	145.62	3.60
63	9198.34	139.16	3.89

with the following mortality probabilities

x	l_x	d_x
58	10000.00	1.81
59	9998.19	1.92
60	9996.27	2.04
61	9994.22	2.18
62	9992.05	2.32
63	9989.73	2.47

[The first decrement is surrender, the second is death.] Using:

(a) UDD in the multiple decrement table.

(b) UDD in the independent decrements.

8. The mortalities for a husband and wife (whose lives are assumed to be independent) aged 62 and 53 respectively, are given in the following tables:

x	l_x	d_x
62	10000.00	5.31
63	9994.69	5.76
64	9988.93	6.25
65	9982.68	6.79
66	9975.89	7.37
67	9968.52	8.01

x	l_x	d_x
53	10000.00	3.03
54	9996.97	3.25
55	9993.72	3.48
56	9990.24	3.74
57	9986.49	4.03
58	9982.47	4.33

The interest rate is $i = 0.03$.

(a) They want to purchase a 5-year joint life insurance policy with a death benefit of \$2,500,000. Annual premiums are payable while both are alive. Calculate the net premium for this policy using the equivalence principle.

(b) They want to purchase a 5-year reversionary annuity, which will provide an annuity to the husband of \$60,000 at the end of each year for the 5-year term if the wife is dead and the husband is alive. Calculate the net premium for this policy using the equivalence principle.

(c) They want to purchase a 5-year last survivor insurance policy, with a death benefit of \$120,000,000. Premiums are payable while either life is alive. Calculate the net premium for this policy using the equivalence principle.

9. A husband is 64; the wife is 73. Their lifetables while both are alive, and the lifetable for the husband if the wife is dead, are given below:

x	l_x	d_x
64	10000.00	6.92
65	9993.08	7.49
66	9985.59	8.12
67	9977.48	8.80
68	9968.68	9.55
69	9959.13	10.36

x	l_x	d_x
73	10000.00	31.73
74	9968.27	34.69
75	9933.58	37.92
76	9895.66	41.45
77	9854.20	45.30
78	9808.91	49.49

x	l_x	d_x
64	10000.00	11.56
65	9988.44	12.56
66	9975.88	13.65
67	9962.23	14.83
68	9947.40	16.12
69	9931.28	17.53

Calculate the probability that the husband survives to the end of the 5-year period. Use the UDD assumption for handling changes to the husband's mortality in the event of the wife's death.

10. A couple want to receive the following:

- While both are alive, they would like to receive a pension of \$90,000 per year.
- If the husband is alive and the wife is not, they would like to receive a pension of \$85,000 per year.
- If the wife is alive and the husband is not, they would like to receive a pension of \$65,000 per year.
- When one dies, if the husband dies first, they would like to receive \$92,000, if the wife dies first, they would like to receive \$120,000.
- When the second one dies, if it is the husband, they would like to receive a benefit of \$65,000; if it is the wife, they would like to receive a benefit of \$93,000.

Construct a combination of insurance and annuity policies that achieve this combination of benefits.

11. A husband aged 52 and wife aged 66 have the following transition intensities:

$$
\begin{aligned}
\mu^{01}_{xy} &= 0.000003y + 0.0000001x \\
\mu^{02}_{xy} &= 0.0000015x + 0.0000004y \\
\mu^{03}_{xy} &= 0.000042 + 0.000013x + 0.000019y \\
\mu^{13}_{x} &= 0.000004x \\
\mu^{23}_{x} &= 0.000003y
\end{aligned}
$$

Which of the following expressions gives the probability that after 7 years, the husband is dead and the wife is alive? Justify your answer.

(i) $\int_0^7 e^{-(0.0015595+0.0020203t+0.0000205t^2)}(0.00003965 + 0.0000039t)\,dt$

(ii) $\int_0^7 e^{-(0.0023614+0.0014475t+0.0000205t^2)}(0.00003465 + 0.0000019t)\,dt$

(iii) $\int_0^7 e^{-(0.0015595+0.0019496t+0.0000170t^2)}(0.00003465 + 0.0000019t)\,dt$

(iv) $\int_0^7 e^{-(0.0009948+0.0020203t+0.0000150t^2)}(0.00003465 + 0.0000019t)\,dt$

12. An individual aged 42 has a current salary of \$76,000 for the coming year. The salary scale is $s_y = 1.05^y$. Estimate the individual's final average salary (average of last 3 years working) assuming the individual retires at exact age 65.

13. An employer sets up a DC pension plan for its employees. The target replacement ratio is 60% of final average salary for an employee who enters the plan at exact age 30, with the following assumptions:

    - At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity for the employee's spouse, valued at 60% of the life annuity.
    - At age 65, the employee is married to someone aged 63.
    - The salary scale is $s_y = 1.04^y$.
    - Mortalities are independent and given by $\mu_x = 0.0000016(1.092)^x$. The value of the life annuity is based on $\delta = 0.045$. This gives $\overline{a}_{65} = 19.63036$, $\overline{a}_{63} = 19.83656$ and $\overline{a}_{65,63} = 18.7867$.
    - A fixed percentage of salary is payable annually in arrear.
    - Contributions earn an annual rate of 7%.

    Calculate the percentage of salary payable annually to achieve the target replacement rate under these assumptions.

14. The salary scale is given in the following table:

y	s_y	y	s_y	y	s_y	y	s_y
30	1.000000	39	1.350398	48	1.845766	57	2.553877
31	1.033333	40	1.397268	49	1.912422	58	2.649694
32	1.067933	41	1.445983	50	1.981785	59	2.749515
33	1.103853	42	1.496620	51	2.053975	60	2.853522
34	1.141149	43	1.549263	52	2.129115	61	2.961903
35	1.179879	44	1.604000	53	2.207337	62	3.074855
36	1.220103	45	1.660921	54	2.288777	63	3.192585
37	1.261887	46	1.720122	55	2.373580	64	3.315310
38	1.305295	47	1.781702	56	2.461894	65	3.443256

An employee aged 42 and 4 months has 12 years of service, and a current salary of \$106,000 (for the coming year). She has a defined benefit pension plan with $\alpha = 0.02$ and S_{Fin} is the average of her last 3 years' salary. The employee's mortality is given by $\mu_x = 0.00000195(1.102)^x$. The pension benefit is payable monthly in advance. The interest rate is $i = 0.05$. This results in $\ddot{a}_{65}^{(12)} = 17.15373$ and ${}_{22.66666667}p_{42.33333333} = 0.9901951$. There is no death benefit, and there are no exits other than death or retirement at age 65.

(a) Calculate the EPV of the accrued benefit using the projected unit method under the assumption that the employee retires at age 65. [Calculate the salary scale at non-integer ages by linear interpolation.]

(b) Calculate the employer's contribution for this employee for the year. [${}_{21.66666667}p_{43.33333333} = 0.9903189$.]

15. The service table is given below:

x	l_x	1	2	3
40	10000.00	118.76	0	0.51
41	9880.73	112.29	0	0.58
42	9767.86	107.16	0	0.65
43	9660.05	101.84	0	0.73
44	9557.49	96.80	0	0.82
45	9459.86	92.02	0	0.93
46	9366.91	87.50	0	1.04
47	9278.37	83.19	0	1.18
48	9193.99	80.11	0	1.32
49	9112.57	75.21	0	1.49
50	9035.87	71.48	0	1.68
51	8962.71	67.92	0	1.89
52	8892.90	64.51	0	2.12
53	8826.26	61.23	0	2.39
54	8762.64	58.07	0	2.69
55	8701.88	55.03	0	3.03
56	8643.83	52.06	0	3.41
57	8588.36	49.18	0	3.84
58	8535.34	46.37	0	4.32
59	8484.64	43.62	0	4.86
60^-	8484.64		1098.84	
60	7385.80	21.70	819.91	5.79
61	6538.40	18.30	611.98	6.38
62	5901.74	10.81	384.29	5.86
63	5500.78	9.14	639.20	6.15
64	4846.29	7.73	351.32	6.10
65^-	4481.14		4481.14	

The salary scale is $s_y = 1.05^y$. The accrual rate is 0.02. The benefit for employees who withdraw is a deferred annual pension with COLA 2%, starting from age 65. For an individual aged 65, we have $\ddot{a}_{65} = 12.85$. The lifetable for an individual who has withdrawn is

x	l_x	d_x
57	10000.00	7.54
58	9992.46	8.22
59	9984.24	8.95
60	9975.29	9.76
61	9965.52	10.65
62	9954.87	11.63
63	9943.25	12.69
64	9930.55	13.86
65	9916.69	15.15

Calculate the EPV of deferred pension benefits made to an individual aged exactly 57, with 16 years of service, whose salary for the past year was $121,000.