

ACSC/STAT 4720, Life Contingencies II

Fall 2016

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Homework Sheet 4

Due: Friday 4th November: 12:30 PM

Basic Questions

1. A life aged 52 has mortality given in the table below. The yield rate is in another table below

x	l_x	d_x	Term (years)	Yield rate
52	10000.00	51.66	1	0.034
53	9948.34	56.61	2	0.032
54	9891.73	62.01	3	0.037
55	9829.71	67.89	4	0.038
56	9761.82	74.28	5	0.038
57	9687.54	81.21		

Calculate the net annual premium for a 5-year term insurance with benefit \$500,000 sold to this life.

2. An insurance company sells N one-year life insurance policies to lives aged 45. The death benefit is \$530,000, payable at the end of the year to lives which die during the year. The company uses $q_{45} = 0.0004$ and $i = 0.06$ to calculate the premium for the policy. This results in a net premium of $530000 \times 0.0004(1.06)^{-1} = \200 .

However, q_{45} is an estimated probability based on past data, and the true value is normally distributed with mean 0.0004 and standard deviation 0.00005. The interest rate cannot be fixed, and the actual interest rate obtained is normally distributed with mean 0.06 and standard deviation 0.01.

Calculate the expected aggregate profit of the policies, and the variance of this aggregate profit. (Calculate the profit at the end of the year.)

3. An insurance company sells a 5-year life-insurance policy with a death benefit of \$450,000 to a life aged 43 for whom the following lifetable is appropriate:

x	l_x	d_x
43	10000.00	17.44
44	9982.56	19.22
45	9963.35	21.17
46	9942.17	23.32
47	9918.85	25.69

The company calculates the annual premium using the current interest rate of $i = 0.05$. This gives a premium of \$910.52. However, the insurance company uses the annual interest rates applicable at the time each year to invest the premiums. These interest rates follow a log-normal distribution with $\mu = 0.05$ and $\sigma = 0.01$. They simulate the following 5 sets of one-year interest rates:

	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
Year 2	4.01%	4.18%	5.22%	5.10%	2.85
Year 3	6.05%	5.26%	6.20%	6.74%	5.72
Year 4	5.47%	5.38%	2.66%	5.03%	4.65
Year 5	3.35%	6.01%	4.32%	3.22%	4.18

- (a) Use these simulated interest rates to estimate the EPV of future loss for this policy.
(b) Construct a 95% confidence interval for the EPV future loss from these simulations.

Standard Questions

4. An insurance company sells 1000 5-year term policies with a death benefit of \$400,000 to lives aged 38 with the following lifetable.

x	l_x	d_x
38	10000.00	13.46
39	9986.54	15.17
40	9971.37	17.09
41	9954.28	19.25
42	9935.03	21.69
43	9913.34	24.43

Using an interest rate of $i = 0.02$, the insurance company sets the premium to \$678.53.

- (a) Assuming the interest rate is correct, calculate the probability that the present value of loss on these policies exceeds \$100,000.
(b) Suppose that the interest rate is uniformly distributed between 0 and $i = 0.04$. Show that the probability that the loss exceeds \$1,000,000 is larger than your answer to part (a). [Hint: it is sufficient to consider the cases where the number of policyholders who die guarantees that the loss exceeds \$1,000,000.]