ACSC/STAT 4720, Life Contingencies II
FALL 2017
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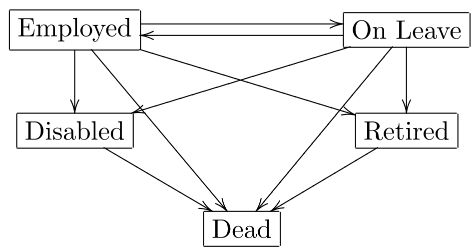
Sample Midterm Examination

This Sample examination has more questions than the actual midterm, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An insurance company is considering a new policy. The policy includes states with the following state diagram:



Which of the following sequences of transitions are possible? (Indicate which parts of the transition sequence are not possible if the sequence is not possible.)

(i) Alive—Disabled—Retired—Dead

(ii) Alive—On Leave—Retired—Dead

(iii) Alive—Retired—On leave—Dead

(iv) Alive—On leave—Alive—Retired—Dead

[5 mins.]

2. Consider a permanent disability model with transition intensities

$$\mu_x^{01} = 0.002 + 0.000005x$$
$$\mu_x^{02} = 0.001 + 0.0000004x^2$$
$$\mu_x^{12} = 0.003 + 0.000004x$$

where State 0 is healthy, State 1 is permanently disabled and State 2 is dead. Write down an expression for the probability that an individual aged 29 is alive but permanently disabled at age 56. [You do not need to evaluate the expression, but should perform basic simplifications on it.] [10 mins.]

3. A disability income model has transition intensities

$$\mu_x^{01} = 0.002$$
$$\mu_x^{10} = 0.001$$
$$\mu_x^{02} = 0.002$$
$$\mu_x^{12} = 0.004$$

State 0 is healthy, State 1 is sick and State 2 is dead. Three actuaries calculate different values for the transition probabilities and benefit values. Which one has calculated plausible values? Justify your answer by explaining what is impossible about the values calculated by the other two actuaries.

Value	Actuary I	Actuary II	Actuary III
${}_2p_{37}^{(00)}$	0.992036	0.992036	0.992036
${}_2p_{37}^{(01)}$	0.003960	0.003968	0.003964
${}_4p_{37}^{(01)}$	0.007857	0.007857	0.007857
${}_4p_{37}^{(02)}$	0.015857	0.008000	0.008000
${}_4p_{37}^{(12)}$	0.008000	0.015857	0.015857
${}_2p_{39}^{(01)}$	0.003960	0.003968	0.003964
${}_2p_{39}^{(11)}$	0.992054	0.992054	0.990054

[10 mins.]

4. A disability income model has the following four states:

State	Meaning
0	Healthy
1	Sick
2	Accidental Death
3	Other Death

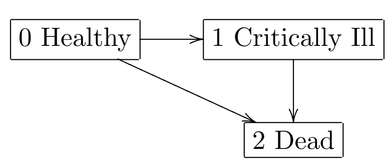
The transition intensities are:

$$\mu_x^{01} = 0.001$$
$$\mu_x^{02} = 0.002$$
$$\mu_x^{03} = 0.001$$
$$\mu_x^{10} = 0.002$$
$$\mu_x^{12} = 0.001$$
$$\mu_x^{13} = 0.003$$

You calculate that the probability that the life is healthy t years from the start of the policy is $0.2113249e^{-0.006732051t} + 0.7886751e^{-0.003267949t}$, and the probability that the life is sick t years from the start of the policy is $0.2886752e^{-0.003267949t} - 0.2886752e^{-0.006732051t}$.

Calculate the premium for a 5-year policy with premiums payable continuously while the life is in the healthy state, which pays no benefits while the life is in the sick state, but pays a benefit of \$200,000 in the event of accidental death and a benefit of \$100,000 in the event of other death. The interest rate is $\delta = 0.03$. [15 mins.]

5. Under a certain model for transition intensities in a critical illness model, with the following transition diagram:



you calculate:

${}_5p_{41}^{00} = 0.866102$	${}_5p_{41}^{01} = 0.0542667$	${}_5p_{41}^{02} = 0.0796309$
$\overline{a}_{41}^{00} = 13.5501$	$\overline{a}_{41}^{01} = 2.48302$	$\overline{a}_{41}^{02} = 8.96688$
$\overline{a}_{46}^{0,0} = 13.1355$	$\overline{a}_{46}^{0,1} = 2.49464$	$\overline{a}_{46}^{0,2} = 9.36984$
$\overline{a}_{46}^{1,1} = 13.2984$	$\overline{a}_{46}^{1,2} = 11.7016$	
$\overline{A}_{41}^{01} = 0.196752$	$\overline{A}_{41}^{02} = 0.358682$	$\overline{A}_{46}^{01} = 0.202971$
$\overline{A}_{46}^{02} = 0.374801$	$\overline{A}_{46}^{12} = 0.468071$	

where 0 is healthy, 1 is critically ill, and 2 is dead. Calculate the premium for a 5-year policy for a life aged 41, with continuous premiums payable while in the healthy state, which pays a benefit $280,000 immediately upon death in the case of death directly from the healthy state and a benefit of $190,000 upon entry to the critically ill state, followed by a further benefit of $140,000 upon death after diagnosis of critical illness. Force of interest is $\delta = 0.04$. [10 mins.]

6. The following is a multiple decrement table giving probabilities of surrender (decrement 1) and death (decrement 2) for a life insurance policy:

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
49	10000.00	235.54	1.46
50	9763.00	222.44	1.55
51	9539.01	210.28	1.65
52	9327.08	198.99	1.77

A life insurance policy has a death benefit of $400,000 payable at the end of the year of death. Premiums are payable at the beginning of each year. Calculate the premium for a 4-year policy sold to a life aged 49 if there is no-payment to policyholders who surrender their policy, and the interest rate is $i = 0.06$.

7. Update the multiple decrement table below

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
58	10000.00	176.04	2.68
59	9823.96	167.67	2.88
60	9656.29	159.84	3.10
61	9496.46	152.50	3.34
62	9343.96	145.62	3.60
63	9198.34	139.16	3.89

with the following mortality probabilities

x	l_x	d_x
58	10000.00	1.81
59	9998.19	1.92
60	9996.27	2.04
61	9994.22	2.18
62	9992.05	2.32
63	9989.73	2.47

[The first decrement is surrender, the second is death.] Using:

(a) UDD in the multiple decrement table.

(b) UDD in the independent decrements.

8. The mortalities for a husband and wife (whose lives are assumed to be independent) aged 62 and 53 respectively, are given in the following tables:

x	l_x	d_x
62	10000.00	5.31
63	9994.69	5.76
64	9988.93	6.25
65	9982.68	6.79
66	9975.89	7.37
67	9968.52	8.01

x	l_x	d_x
53	10000.00	3.03
54	9996.97	3.25
55	9993.72	3.48
56	9990.24	3.74
57	9986.49	4.03
58	9982.47	4.33

The interest rate is $i = 0.03$.

(a) They want to purchase a 5-year joint life insurance policy with a death benefit of \$2,500,000. Annual premiums are payable while both are alive. Calculate the net premium for this policy using the equivalence principle.

(b) They want to purchase a 5-year reversionary annuity, which will provide an annuity to the husband of \$60,000 at the end of each year for the 5-year term if the wife is dead and the husband is alive. Calculate the net premium for this policy using the equivalence principle.

(c) They want to purchase a 5-year last survivor insurance policy, with a death benefit of \$120,000,000. Premiums are payable while either life is alive. Calculate the net premium for this policy using the equivalence principle.

9. A husband is 64; the wife is 73. Their lifetables while both are alive, and the lifetable for the husband if the wife is dead, are given below:

x	l_x	d_x
64	10000.00	6.92
65	9993.08	7.49
66	9985.59	8.12
67	9977.48	8.80
68	9968.68	9.55
69	9959.13	10.36

x	l_x	d_x
73	10000.00	31.73
74	9968.27	34.69
75	9933.58	37.92
76	9895.66	41.45
77	9854.20	45.30
78	9808.91	49.49

x	l_x	d_x
64	10000.00	11.56
65	9988.44	12.56
66	9975.88	13.65
67	9962.23	14.83
68	9947.40	16.12
69	9931.28	17.53

Calculate the probability that the husband survives to the end of the 5-year period. Use the UDD assumption for handling changes to the husband's mortality in the event of the wife's death.

10. A couple want to receive the following:

- While both are alive, they would like to receive a pension of \$90,000 per year.
- If the husband is alive and the wife is not, they would like to receive a pension of \$85,000 per year.
- If the wife is alive and the husband is not, they would like to receive a pension of \$65,000 per year.
- When one dies, if the husband dies first, they would like to receive \$92,000, if the wife dies first, they would like to receive \$120,000.
- When the second one dies, if it is the husband, they would like to receive a benefit of \$65,000; if it is the wife, they would like to receive a benefit of \$93,000.

Construct a combination of insurance and annuity policies that achieve this combination of benefits.

11. A husband aged 52 and wife aged 66 have the following transition intensities:

$$\begin{aligned}
\mu_{xy}^{01} &= 0.000003y + 0.0000001x \\
\mu_{xy}^{02} &= 0.0000015x + 0.0000004y \\
\mu_{xy}^{03} &= 0.000042 + 0.000013x + 0.000019y \\
\mu_{x}^{13} &= 0.000004x \\
\mu_{x}^{23} &= 0.000003y
\end{aligned}$$

Which of the following expressions gives the probability that after 7 years, the husband is dead and the wife is alive? Justify your answer.

(i) $\int_0^7 e^{-(0.0015595+0.0020203t+0.0000205t^2)}(0.00003965 + 0.0000039t)\,dt$

(ii) $\int_0^7 e^{-(0.0023614+0.0014475t+0.0000205t^2)}(0.00003465 + 0.0000019t)\,dt$

(iii) $\int_0^7 e^{-(0.0015595+0.0019496t+0.0000170t^2)}(0.00003465 + 0.0000019t)\,dt$

(iv) $\int_0^7 e^{-(0.0009948+0.0020203t+0.0000150t^2)}(0.00003465 + 0.0000019t)\,dt$

12. A life aged 38 wants to buy a 3-year term insurance policy. A life-table based on current-year mortality is:

x	l_x	d_x
38	10000.00	5.00
39	9995.00	5.14
40	9989.86	5.30
41	9984.56	5.47
42	9979.09	5.67
43	9973.42	5.87

The insurance company uses a single-factor scale function $q(x,t) = q(x,0)(1-\phi_x)^t$ to model changes in mortality. The insurance company uses the following values for ϕ_x:

x	ϕ_x
38	0.03
39	0.025
40	0.025
41	0.02
42	0.015
43	0.02

Calculate $A^{1}_{38:\overline{3}|}$ at interest rate $i = 0.06$, taking into account the change in mortality.

13. The following lifetable applied in 2016:

x	l_x	d_x
55	10000.00	10.63
56	9989.37	11.30
57	9978.07	12.02
58	9966.05	12.80
59	9953.25	13.66
60	9939.59	14.60

An insurance company uses the following mortality scale based on both age and year:

	t					
x	2017	2018	2019	2020	2021	2022
55	0.01	0.015	0.015	0.02	0.02	0.015
56	0.03	0.03	0.025	0.02	0.015	0.02
57	0.02	0.03	0.03	0.025	0.02	0.015
58	0.025	0.03	0.025	0.015	0.015	0.02
59	0.015	0.02	0.015	0.01	0.015	0.01
60	0.02	0.015	0.01	0.015	0.02	0.025

Use this mortality scale to calculate $A^{1}_{55:\overline{4}|}$ at interest rate $i = 0.03$.

14. A pensions company has the current mortality scale for 2017:

x	$\phi(x, 2017)$	$\left.\frac{d\phi(x,t)}{dt}\right\|_{x,t=2017}$	$\left.\frac{d\phi(x+t,t)}{dt}\right\|_{x,t=2017}$
51	0.016389776	0.00054272913	−0.0015000971
52	0.018738397	−0.00107674028	0.0012410504
53	0.028229446	0.00120650853	−0.0002976607
54	0.028011768	−0.00109930339	−0.0004183465
55	0.014334489	−0.00194027424	0.0023952205
56	0.016770205	0.00271342277	−0.0053102487

Mortality in 2016 is given in the following lifetable.

x	l_x	d_x
51	10000.00	15.29
52	9984.71	16.44
53	9968.27	17.70
54	9950.56	19.09
55	9931.48	20.60
56	9910.88	22.26

The company assumes that from 2030 onwards, we will have $\phi(x, t) = 0.01$ for all x and t. Calculate $q(54, 2018)$ using the average of age-based and cohort-based effects.

15. An insurance company uses a Lee-Carter model and fits the following parameters:

$$c = -0.6 \qquad \sigma_k = 1.4 \qquad K_{2017} = -4.83$$

And the following values of α_x and β_x:

x	α_x	β_x
34	-5.314675	0.2697754
35	-5.234098	0.2504377
36	-5.043921	0.1782635
37	-4.892803	0.2889967
38	-4.637988	0.1460634
39	-4.413315	0.1174245
40	-4.261060	0.2078267

The insurance company simulates the following values of Z_t:

t	Z_t
2018	0.2525295
2019	-0.6276655
2020	-0.6007807

Using these simulated values, calculate the probability that a life aged exactly 36 at the start of 2017 dies within the next 4 years.

16. An insurance company uses a Lee-Carter model. One actuary fits the following parameters:

$$c = -0.13 \qquad \sigma_k = 0.9 \qquad K_{2017} = -1.70 \qquad \alpha_{52} = -4.45 \qquad \beta_{52} = 0.49$$

A second actuary fits the parameters

$$c = -0.14 \qquad \sigma_k = 0.8 \qquad K_{2017} = -1.40 \qquad \alpha_{52} = -4.94 \qquad \beta_{52} = 0.37$$

The insurance company sets its life insurance premiums for 2025 so that under the first actuary's model, it has a 95% chance of an expected profit. What is the probability that these premiums lead to an expected profit under the second actuary's model?

17. An insurance company uses a Cairns-Blake-Dowd model with the following parameters:

$$K^{(1)}_{2017} = -3.29 \qquad K^{(2)}_{2017} = 0.38 \qquad c^{(1)} = -0.17 \qquad c^{(2)} = 0.01$$
$$\sigma_{k_1} = 0.5 \qquad \sigma_{k_2} = 0.08 \qquad \rho = 0.3 \qquad \overline{x} = 47$$

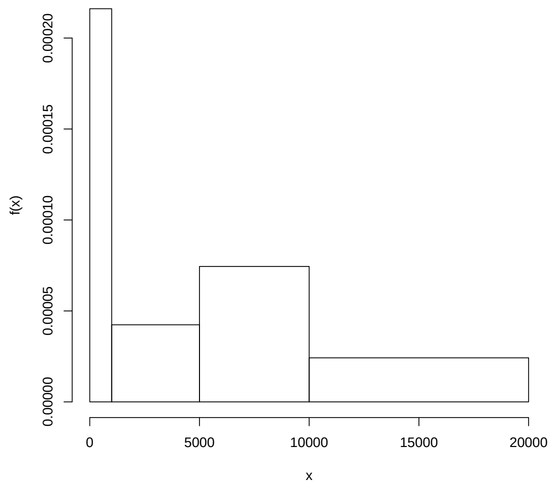
What is the probability that the mortality for an individual currently (in 2017) aged 39 will be higher in 2025 than in 2030?

18. For the following dataset:

```
0.2 0.2 0.4 0.7 1.8 2.1 2.3 3.0 3.5 3.9 4.1 4.2 4.6 5.1 5.7 6.6
8.2 11.4
```

Calculate a Nelson-Åalen estimate for the probability that a random sample is more than 2.7.

19. The histogram below is obtained from a sample of 8,000 claims.



Which interval included most claims?

20. An insurance company collects the following data on insurance claims:

Claim Amount	Number of Policies
Less than \$5,000	232
\$5,000–\$20,000	147
\$20,000–\$100,000	98
More than \$100,000	23

The policy currently has no deductible and a policy limit of \$100,000. The company wants to determine how much would be saved by introducing a deductible of \$2,000 and a policy limit of \$50,000. Using the ogive to estimate the empirical distribution, how much would the expected claim amount be reduced by the new deductible and policy limit?

21. An insurance company collects the following claim data (in thousands):

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	0.8	-	8	0.5	-	5	15	2.0	-	5
2	0	1.3	-	9	1.0	1.2	-	16	2.0	-	10
3	0	-	20	10	1.0	-	15	17	2.0	2.4	-
4	0	4.4	-	11	1.0	1.8	-	18	2.0	-	5
5	0	-	10	12	1.0	-	10	19	2.0	11.6	-
6	0.5	1.4	-	13	1.0	6.3	-	20	5.0	-	15
7	0.5	1.8	-	14	2.0	4.9.	-	21	5.0	5.9	-

Using a Kaplan-Meier product-limit estimator:

(a) estimate the probability that a random loss exceeds 3.

(b) Use Greenwood's approximation to obtain a 95% confidence interval for the probability that a random loss exceeds 3, based on the Kaplan-Meier estimator, using a normal approximation.

(c) Use Greenwood's approximation to find a log-transformed confidence interval for the probability that a random loss exceeds 3.

22. An insurance company records the following data in a mortality study:

entry	death	exit	entry	death	exit	entry	death	exit
51.3	-	58.4	56.5	-	58.2	55.3	-	59.9
54.7	-	59.7	54.7	-	59.8	53.3	59.1	
53.8	-	58.5	57.9	-	61.3	56.7	58.4	-
57.3	-	58.3	58.0	-	59.3	52.4	58.9	-
52.8	-	60.6	58.4	-	59.8	57.7	58.8	-
58.7	-	59.5	53.0	-	58.3	58.3	60.4	-
53.3	-	62.4	53.1	-	60.1	58.1	58.4	-

Estimate the probability of an individual currently aged exactly 58 dying within the next year using:

(a) the exact exposure method.

(b) the actuarial exposure method.

23. Using the following table:

Age	No. at start	enter	die	leave	No. at next age
48	26	43	2	13	54
49	54	39	7	17	69
50	69	46	14	28	73
51	73	22	13	44	38

Estimate the probability that an individual aged 49 withdraws from the policy within the next two years, conditional on surviving to the end of those two years.