# ACSC/STAT 4720, Life Contingencies II Fall 2017 

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Homework Sheet 4
Due: Friday 20th October: 12:30 PM

## Basic Questions

1. A disability income insurance company collects the following claim data (in thousands):

| $i$ | $d_{i}$ | $x_{i}$ | $u_{i}$ | $i$ | $d_{i}$ | $x_{i}$ | $u_{i}$ | $i$ | $d_{i}$ | $x_{i}$ | $u_{i}$ |
| :---: | :---: | ---: | ---: | :--- | :--- | ---: | ---: | :--- | :--- | :--- | ---: |
| 1 | 0 | 1.9 | - | 8 | 0.5 | 0.6 | - | 15 | 2.0 | - | 5 |
| 2 | 0 | - | 5 | 9 | 0.5 | 1.3 | - | 16 | 2.0 | 4.4 | - |
| 3 | 0 | 2.1 | - | 10 | 0.5 | 0.7 | - | 17 | 2.0 | 4.5 | - |
| 4 | 0 | 0.3 | - | 11 | 0.5 | 2.5 | - | 18 | 2.0 | 3.9 | - |
| 5 | 0 | 0.1 | - | 12 | 1.0 | 3.5 | - | 19 | 5.0 | 6.3 | - |
| 6 | 0 | 0.1 | - | 13 | 1.0 | - | 5 | 20 | 5.0 | 7.0 | - |
| 7 | 0 | 2.1 | - | 14 | 1.0 | 5.0 | - | 21 | 5.0 | 7.9 | - |

Using a Kaplan-Meier product-limit estimator:
(a) estimate the probability that a random loss exceeds 3.4.
(b) estimate the median of the distribution.
(c) Use a Nelson-Åalen estimator to estimate the median of the distribution.
2. For the data in Question 1, use Greenwood's approximation to obtain a $95 \%$ confidence interval for the probability that a random loss exceeds 3.4, based on the Kaplan-Meier estimator.
(a) Using a normal approximation
(b) Using a log-transformed confidence interval.
3. An insurance company records the following data in a mortality study:

| entry | death | exit | entry | death | exit | entry | death | exit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 68.4 | 71.0 | - | 69.8 | - | 73.7 | 69.1 | - | 72.1 |
| 68.3 | 71.4 | - | 68.4 | 72.8 | - | 68.6 | - | 72.3 |
| 69.1 | 73.8 | - | 68.7 | - | 71.4 | 71.0 | - | 71.9 |
| 70.5 | - | 72.6 | 70.0 | - | 72.1 | 70.3 | - | 71.0 |
| 69.3 | - | 72.8 | 70.3 | - | 72.0 | 68.6 | 72.1 | - |
| 69.0 | 73.1 | - | 70.6 | - | 73.1 | 68.7 | - | 72.6 |
| 70.6 | - | 71.3 | 70.2 | - | 71.3 | 69.7 | - | 73.8 |
| 69.7 | - | 72.4 | 71.0 | 72.9 | - | 70.6 | - | 73.5 |
| 68.5 | - | 72.3 | 69.2 | - | 71.8 | 70.7 | 72.3 | - |
| 70.6 | - | 71.4 | 70.4 | - | 71.7 | 69.6 | - | 72.3 |
| 69.4 | 71.4 | - | 68.3 | - | 73.4 | 68.2 | - | 72.8 |

Estimate the probability of an individual currently aged exactly 71 dying within the next year using:
(a) the exact exposure method.
(b) the actuarial exposure method.
4. Using the following table:

| Age | No. at start | enter | die | leave | No. at next age |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 55 | 0 | 28 | 4 | 10 | 14 |
| 56 | 14 | 31 | 3 | 14 | 28 |
| 57 | 28 | 21 | 6 | 24 | 19 |
| 58 | 19 | 38 | 1 | 42 | 14 |
| 59 | 14 | 29 | 2 | 41 | 0 |

Estimate the probability that an individual aged 58 withdraws from the policy within the next year, conditional on surviving to the end of the year.

## Standard Questions

5. For the study in Question 3, use the actuarial exposure method, and assume that the number of deaths follows a Poisson distribution with mean exposure times probability of dying to find a $95 \%$ confidence interval for $q_{71}$.
