ACSC/STAT 4720, Life Contingencies II
Fall 2016

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Homework Sheet 6

Due: Friday 24th November: 12:30 PM

Basic Questions

1. An individual aged 45 has a current salary of \$67,000. The salary scale is $s_y = 1.05^y$. Estimate the individual's final average salary (average of last 3 years working) assuming the individual retires at exact age 65.

2. An employer sets up a DC pension plan for its employees. The target replacement ratio is 70% of final average salary for an employee who enters the plan at exact age 26, with the following assumptions:

   - At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity for the employee's spouse, valued at 70% of the life annuity.
   - At age 65, the employee is married to someone aged 62.
   - The salary scale is $s_y = 1.06^y$.
   - Mortalities are independent and given by $\mu_x = 0.0000017(1.101)^x$.
   - A fixed percentage of salary is payable monthly in arrear.
   - Contributions earn an annual rate of 5%.
   - The value of the life annuity is based on $\delta = 0.04$.

   Calculate the percentage of salary payable monthly to achieve the target replacement rate under these assumptions. [You may use numerical integration to compute the value of the annuities.]

3. The salary scale is given in the following table:

y	s_y	y	s_y	y	s_y	y	s_y
30	1.000000	39	1.350398	48	1.845766	57	2.553877
31	1.033333	40	1.397268	49	1.912422	58	2.649694
32	1.067933	41	1.445983	50	1.981785	59	2.749515
33	1.103853	42	1.496620	51	2.053975	60	2.853522
34	1.141149	43	1.549263	52	2.129115	61	2.961903
35	1.179879	44	1.604000	53	2.207337	62	3.074855
36	1.220103	45	1.660921	54	2.288777	63	3.192585
37	1.261887	46	1.720122	55	2.373580	64	3.315310
38	1.305295	47	1.781702	56	2.461894	65	3.443256

   An employee aged 46 and 2 months has 8 years of service, and a current salary of \$83,000 (for the coming year). She has a defined benefit pension plan with $\alpha = 0.015$ and S_{Fin} is the average of her last 3 years' salary. The employee's mortality is given by $\mu_x = 0.00000104(1.111)^x$. The pension benefit is payable monthly in advance. The interest rate is $i = 0.06$. [This gives $\ddot{a}_{65}^{(12)} = 14.98951$.] Calculate the EPV of the accrued benefit under the assumption that the employee retires at age 65.

Standard Questions

4. An employee aged 58 has been working with a company for 23 years. The employee's salary last year was \$94,000. The salary scale is the same as for Question 3. The service table is given below:

t	${}_tp^{(00)}$	1	2	3
0	10000.00	48.48	0	6.48
1	9945.03	45.29	0	7.29
2^-	9892.45		1187.53	
2	8704.92	25.72	142.14	6.86
3	8530.20	21.69	128.54	7.56
4^-	8372.40		1426.64	
4	6945.76	12.81	455.53	6.95
5	6470.47	10.83	757.69	7.29
6	5694.66	9.16	416.44	7.23
7^-	5261.82		5311.82	

Mortality follows a Gompertz model with $B = 0.0000012$ and $C = 1.1$. If the member withdraws, he receives a defered pension starting from age 65, with 2% COLA. The death benefit of the plan is three times the employee's final average salary if the employee is still working at the time of death. If the employee has withdrawn, the death benefit is three times final average salary with COLA of 2%. The accrual rate for the pension is 0.02. Pension payments are made annually in advance. The interest rate is $i = 0.04$.

Calculate the EPV of the accrued benefit. [You may assume that events happen in the middle of each year.]

You are given the following values:

x	$\ddot{a}_x$
60	20.85185
60.5	20.75652
61.5	20.56113
62	20.46258
62.5	20.35935
63.5	20.15106
64.5	19.93613
65	19.83053

5. An individual aged 43 has 4 years of service, and last year's salary was \$74,000. The salary scale is $s_y = 1.05^y$. The accrual rate is 0.01. The interest rate is $i = 0.05$. There is no death benefit, and no exits other than death or retirement at age 65. The pension benefit is payable annually in advance. Mortality follows a Gompertz law with $B = 0.000004$ and $C = 1.09$. You are given that $\ddot{a}_{65} = 18.04168$. Calculate this year's employer contribution to the plan using

   (a) The projected unit method.

   (b) The traditional unit method.