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ACSC/STAT 4720, Life Contingencies II
Fall 2017
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Homework Sheet 7
Due: Friday 1st December: 12:30 PM

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Basic Questions

1. An insurance company sells a 5-year annual life insurance policy to a life aged 54, for whom the lifetable below is appropriate.

x	l_x	d_x
54	10000.00	12.23
55	9987.77	13.47
56	9974.30	14.82
57	9959.48	16.31
58	9943.16	17.95

The annual gross premium is \$1,320. Initial expenses are \$170 plus 30% of the first premium. The death benefits are \$800,000. Renewal costs are 3% of each subsequent premium. The interest rate is $i = 0.05$

(a) Calculate the expected net cash-flows associated with this policy (assuming no reserve). [This is the profit vector for the policy.]

(b) Which of the following is the internal rate of return of the policy:

(i) $i = 0.12429$

(ii) $i = 0.17937$

(iii) $i = 0.23581$

(iv) $i = 0.24836$

2. An insurance company sells a 5-year annual life insurance policy to a life aged 32, for whom the lifetable below is appropriate.

x	l_x	d_x
32	10000.00	2.14
33	9997.86	2.34
34	9995.52	2.56
35	9992.96	2.81
36	9990.15	3.08

The annual gross premium is \$190. Initial expenses are \$60 plus 20% of the first premium. The death benefits are \$680,000. Renewal costs are 2% of each subsequent premium. The interest rate is $i = 0.06$. Reserves are calculated on the basis $i = 0.04$, with mortality following the table.

(a) Calculate the reserves.

(b) Calculate the profit signature.

(c) Calculate the profit margin at a risk discount rate of $i = 0.08$.

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3. For the policy in Question 2:

   (a) Calculate the reserves and profit signature for a general premium. [You may assume that P is such that the reserves are zero in Years 1 and 2.]

   (b) Calculate the premium that gives an internal rate of return of $i = 0.12$.

4. For a 5-year term insurance policy sold to a life aged 39, with the following lifetable:

x	l_x	d_x
39	10000.00	9.64
40	9990.36	10.71
41	9979.65	11.90
42	9967.75	13.23
43	9954.52	14.70

   an actuary performs the following profit test without reserves:

Year	Premium	Expenses	Interest	Expected Death Benefits	Pr_t
0		1500			-1500
1	900	0	36.00	771.00	165.00
2	900	18	35.28	857.90	59.38
3	900	18	35.28	954.19	-36.91
4	900	18	35.28	1061.76	-144.48
5	900	18	35.28	1181.36	-264.08

   Calculate the reserves needed to ensure that all cash flows are non-negative.

   # Standard Questions

5. An insurer sells a 5-year disability income protection policy for a life aged 42. The transition probabilities are given in the following table:

x	p_x^{01}	p_x^{02}	p_x^{10}	p_x^{12}
42	0.002136	0.001426	0.126260	0.082503
43	0.002186	0.001497	0.123351	0.087253
44	0.002241	0.001584	0.120121	0.091034
45	0.002299	0.001715	0.116980	0.096115
46	0.002368	0.001860	0.113402	0.103358

   The probability of being in each state at the start of each year is

t	$_tp_{42}^{00}$	$_tp_{42}^{01}$
1	0.996438	0.002136
2	0.993031596582	0.003864363324
3	0.989697441912	0.00527376749426
4	0.986341721701	0.00642526842903
5	0.982900107192	0.00736818444134

   The policy pays a benefit of \$45,000 at the end of any year if the life is disabled at that time (State 1), and pays a death benefit of \$350,000 at the end of the year if the life is dead (State 2). The interest rate is $i = 0.07$. Initial expenses are \$400 plus 20% of the first premium. Renewal expenses are 2% of each subsequent

premium. The premium is \$1,460 at the start of each year. Use a profit test to calculate the reserves for each year in each state using a reserve rate of $i = 0.06$ and calculate the profit margin at a risk discount rate of $i = 0.08$.