# ACSC/STAT 4720, Life Contingencies II <br> Fall 2018 <br> Toby Kenney <br> Homework Sheet 1 <br> Due: Friday 28th September: 12:30 PM 

## Basic Questions

1. An CCRC is developing a model for its care costs. The community has four levels of care: Independent Living Unit, Assisted Living Unit, Skilled Nursing Facility, and Memory Care Unit. The transition diagram is shown below:


Which of the following sequences of transitions are possible? (Indicate which parts of the transition sequence are not possible if the sequence is not possible.)
(i) ILU-SNF (long-term)- ALU-Dead
(ii) ILU-ALU-SNF (short-term)-ALU
(iii) ILU-MCU-ALU-Dead
(iv) ILU-SNF (short-term)-ILU-ALU
(v) ILU-MCU-SNF (long-term)-Dead
2. Consider a permanent disability model with transition intensities

$$
\begin{aligned}
& \mu_{x}^{01}=0.001+0.000003 x \\
& \mu_{x}^{02}=0.001+0.000004 x \\
& \mu_{x}^{12}=0.004+0.000002 x
\end{aligned}
$$

where State 0 is healthy, State 1 is permanently disabled and State 2 is dead.
(a) Calculate the probability that a healthy individual aged 27 is still healthy at age 44 .
(b) Calculate the probability that a healthy individual aged 33 is dead by age 56 .
3. Under a disability income model with transition intensities

$$
\begin{aligned}
& \mu_{x}^{01}=0.002 \\
& \mu_{x}^{10}=0.004 \\
& \mu_{x}^{02}=0.001 \\
& \mu_{x}^{12}=0.006
\end{aligned}
$$

calculate the probability that a healthy individual has some period of disability within the next 6 years. [State 0 is healthy, State 1 is sick and State 2 is dead.]
4. Under a critical illness model with transition intensities at age $x$ given by:

$$
\begin{aligned}
& \mu_{x}^{01}=0.001+0.000006 x \\
& \mu_{x}^{02}=0.002 \\
& \mu_{x}^{12}=0.12
\end{aligned}
$$

calculate the premium for a whole life policy sold to a life aged 35 with premiums payable continuously while the life is in the healthy state, which pays a death benefit of $\$ 130,000$ upon entry into state 2 , and a benefit of $\$ 120,000$ upon entry into state 1 , sold to a life in the healthy state (state 0 ). The interest rate is $\delta=0.04$ [State 0 is healthy, State 1 is sick and State 2 is dead.]
5. An insurer offers a life insurance policy with an additional benefit for accidental death. The possible exits from this policy are surrender, death (accident) and death (other). The transition intensities are

$$
\begin{aligned}
& \mu_{x}^{01}=0.002+0.000001 x \\
& \mu_{x}^{03}=0.001+0.000006 x \\
& \mu_{x}^{02}=0.004-0.000002 x
\end{aligned}
$$

Calculate the probability that an individual aged 34 dies in an accident before age 72 . [State 0 is in force, State 1 is surrender, State 2 is death (accident) and State 3 is death (other).]

## Standard Questions

6. An insurance company is developing a new model for transition intensities in a disability income model. Under these transition intensities it calculates

$$
\left.\begin{array}{rlrl}
\bar{A}_{34}^{02} & =0.217118 & \bar{A}_{49}^{02} & =0.25344 \\
\bar{a}_{34}^{00} & =12.0453 & \bar{a}_{49}^{00} & =11.2778 \\
{ }_{15} p_{34}^{00} & =0.723952 & 15 p_{34}^{01} & =0.0633742
\end{array} \begin{array}{c}
\bar{A}_{49}^{12}
\end{array}=0.07774320 . \bar{a}_{49}^{10}=0.033278\right)
$$

Calculate the premium for a 15 -year policy for a life aged 34 , with continuous premiums payable while in the healthy state, which pays a continuous benefit while in the sick state, at a rate of $\$ 120,000$ per year, and pays a death benefit of $\$ 700,000$ immediately upon death. [Hint: to calculate $A_{x}^{02}$, consider how to extend the equation $\bar{a}_{x}=\frac{1-\bar{A}_{x}}{\delta}$ to the multiple state case by combining states 0 and 1.]

