ACSC/STAT 4720, Life Contingencies II Fall 2018

Toby Kenney Homework Sheet 1 Due: Friday 28th September: 12:30 PM

Basic Questions

1. An CCRC is developing a model for its care costs. The community has four levels of care: Independent Living Unit, Assisted Living Unit, Skilled Nursing Facility, and Memory Care Unit. The transition diagram is shown below:



Which of the following sequences of transitions are possible? (Indicate which parts of the transition sequence are not possible if the sequence is not possible.)

- (i) ILU–SNF (long-term)– ALU–Dead
- (ii) ILU–ALU–SNF (short-term)–ALU
- (iii) ILU–MCU–ALU–Dead
- (iv) ILU–SNF (short-term)–ILU–ALU
- (v) ILU–MCU–SNF (long-term)–Dead
- 2. Consider a permanent disability model with transition intensities

$$\begin{split} \mu_x^{01} &= 0.001 + 0.000003x \\ \mu_x^{02} &= 0.001 + 0.000004x \\ \mu_x^{12} &= 0.004 + 0.000002x \end{split}$$

where State 0 is healthy, State 1 is permanently disabled and State 2 is dead.

- (a) Calculate the probability that a healthy individual aged 27 is still healthy at age 44.
- (b) Calculate the probability that a healthy individual aged 33 is dead by age 56.
- 3. Under a disability income model with transition intensities

$$\mu_x^{01} = 0.002$$
$$\mu_x^{10} = 0.004$$
$$\mu_x^{02} = 0.001$$
$$\mu_x^{12} = 0.006$$

calculate the probability that a healthy individual has some period of disability within the next 6 years. [State 0 is healthy, State 1 is sick and State 2 is dead.]

4. Under a critical illness model with transition intensities at age x given by:

$$\begin{split} \mu_x^{01} &= 0.001 + 0.000006x \\ \mu_x^{02} &= 0.002 \\ \mu_x^{12} &= 0.12 \end{split}$$

calculate the premium for a whole life policy sold to a life aged 35 with premiums payable continuously while the life is in the healthy state, which pays a death benefit of \$130,000 upon entry into state 2, and a benefit of \$120,000 upon entry into state 1, sold to a life in the healthy state (state 0). The interest rate is $\delta = 0.04$ [State 0 is healthy, State 1 is sick and State 2 is dead.]

5. An insurer offers a life insurance policy with an additional benefit for accidental death. The possible exits from this policy are surrender, death (accident) and death (other). The transition intensities are

$$\mu_x^{01} = 0.002 + 0.000001x$$

$$\mu_x^{03} = 0.001 + 0.000006x$$

$$\mu_x^{02} = 0.004 - 0.000002x$$

Calculate the probability that an individual aged 34 dies in an accident before age 72. [State 0 is in force, State 1 is surrender, State 2 is death (accident) and State 3 is death (other).]

Standard Questions

6. An insurance company is developing a new model for transition intensities in a disability income model. Under these transition intensities it calculates

$$\overline{A}_{34}^{02} = 0.217118 \qquad \overline{A}_{49}^{02} = 0.25344 \qquad \overline{A}_{49}^{12} = 0.0777432 \\ \overline{a}_{34}^{00} = 12.0453 \qquad \overline{a}_{49}^{00} = 11.2778 \qquad \overline{a}_{49}^{10} = 0.033278 \\ {}_{15}p_{34}^{00} = 0.723952 \qquad {}_{15}p_{34}^{01} = 0.0633742 \qquad \delta = 0.05 \\$$

Calculate the premium for a 15-year policy for a life aged 34, with continuous premiums payable while in the healthy state, which pays a continuous benefit while in the sick state, at a rate of \$120,000 per year, and pays a death benefit of \$700,000 immediately upon death. [Hint: to calculate A_x^{02} , consider how to extend the equation $\bar{a}_x = \frac{1-\bar{A}_x}{\delta}$ to the multiple state case by combining states 0 and 1.]