

Suppose X follows a mixture of Poisson distributions with mean Λ where $\Lambda = E^U$ and U has density function $\pi(u)$.

Direct attempts to estimate the distribution of U tend not to converge. To avoid this problem, we take the moments of the zero-truncated distribution. That is, we try to estimate $\mu_n = \mathbb{E}(U^n | X > 0)$. This is reasonably straightforward, since we have that $X|U, X > 0$ follows a zero-truncated Poisson distribution with mean e^U . The n th raw moment of a zero-truncated Poisson distribution with mean λ is

$$\begin{aligned}
& \frac{e^{-\lambda}}{1 - e^{-\lambda}} \sum_{i=1}^{\infty} \frac{i^n \lambda^i}{i!} \\
&= \frac{e^{-\lambda}}{1 - e^{-\lambda}} \sum_{i=1}^{\infty} \sum_{j=1}^n \left\{ \begin{matrix} n \\ j \end{matrix} \right\} \frac{i(i-1) \cdots (i-j+1) \lambda^i}{i!} \\
&= \frac{e^{-\lambda}}{1 - e^{-\lambda}} \sum_{i=1}^{\infty} \sum_{j=1}^n \left\{ \begin{matrix} n \\ j \end{matrix} \right\} \frac{\lambda^i}{(i-j)!} \\
&= \frac{e^{-\lambda}}{1 - e^{-\lambda}} \sum_{i=1}^{\infty} \sum_{j=1}^n \left\{ \begin{matrix} n \\ j \end{matrix} \right\} \lambda^j \frac{\lambda^{i-j}}{(i-j)!} \\
&= \frac{1}{1 - e^{-\lambda}} \sum_{j=1}^n \left\{ \begin{matrix} n \\ j \end{matrix} \right\} \lambda^j
\end{aligned}$$