Summary of Theorems in the Course

Angle Theorems

- Angles on a line add up to $180°$. Conversely, if $\angle ABC = 180°$, then A, B and C are colinear.
- Corresponding angles are equal. Conversely, if the angles are equal, then the lines are parallel.
- Alternate angles are equal. Conversely, if $\angle ABC = \angle BAD$ are equal, then the lines DA and BC are parallel.
- Vertically opposite angles are equal.
- Angles in a triangle add up to $180°$.

Similar Triangles

Two triangles ABC and $A'B'C'$ are congruent if:

- (SSS) $AB = A'B'$, $AC = A'C'$ and $BC = B'C'$.
- (SAS) $AB = A'B'$, $AC = A'C'$ and $\angle BAC = \angle B'A'C'$.
- (ASA) $AB = A'B'$, $\angle ABC = \angle A'B'C'$ and $\angle BAC = \angle B'A'C'$.

Two triangles are similar if:

- (SSS) $\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$.
- (SAS) $\frac{AB}{A'B'} = \frac{AC}{A'C'}$ and $\angle BAC = \angle B'A'C'$.
- (AAA) $\angle ABC = \angle A'B'C'$ and $\angle ACB = \angle A'C'B'$ (and therefore $\angle BAC = \angle B'A'C'$).

Circle Theorems

- The angle subtended at the centre is twice the angle subtended at a point on the circle.
- Angles in the same segment are equal. Conversely, if $\angle ACB = \angle ADB$ and C and D are on the same side of the line AB, then A, B, C, and D are concyclic.
- Opposite angles in a cyclic quadrilateral sum to $180°$. Conversely, if the opposite angles of a quadrilateral sum to $180°$, then the quadrilateral is cyclic.

- The angle in a semicircle is a right angle. Conversely, if $\angle ABC = 90°$, then AC is a diameter of the circumcircle.
- The tangent at a point is perpendicular to the radius.
- Alternate segment theorem: The angle between a tangent and a chord AB is the angle subtended by the arc AB at a point on the circle on the side opposite the tangent.
- Tangents from a point have equal length.
- Intersecting chord theorem: If A, B, C and D are points on the circle, and the lines AB and CD meet at X, then $AX.BX = CX.DX$.
- Intersecting chord and tangent theorem: If A, B and C are points on a circle, and the tangent at C meets the line AB at a point P outside the circle, then $AP.BP = CP^2$.
- Given two circles, the set of points that have the same power with respect to both circles is a line perpendicular to the line between their centres. It is called the radical axis of the two circles.
- Radical axis theorem: Given 3 circles whose centres are not colinear, their radical axes are concurrent (all pass through a single point, called the radical centre of the circles).

Triangle Theorems

- The three medians of the triangle are concurrent. The point G where they meet is called the centroid. G divides each of the medians in the ratio $2:1$.
- The perpendicular bisectors of the three sides of a triangle are concurrent. The point O where they meet is called the circumcentre. It is the centre of the unique circle passing through all three vertices. The radius of this circle is called the circumradius.
- The three altitudes of a triangle are concurrent. The point H where they meet is called the orthocentre of the triangle.
- The three angle bisectors of a triangle are concurrent. The point I where they meet is called the incentre. It is the centre of the circle that is internally tangent to all three sides of the triangle. This circle is called the incircle. Its radius is called the inradius. The angle bisector at one vertex and the external angle bisectors at the other two vertices are also concurrent. The point where they meet is called an excentre. It is the centre of a circle tangent to one side internally, and tangent to the other two sides extended.

- Angle bisector theorem: If ABC is a triangle, and D is the point on BC such that AD is the angle bisector of $\angle BAC$, then $\frac{BD}{CD} = \frac{AB}{AC}$.
- Sine rule: If ABC is a triangle with angles A, B and C, and side lengths a, b and c, and circumradius R then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.
- Cosine rule: If ABC is a triangle with angles A, B and C, and side lengths a, b and c, then $a^2 = b^2 + c^2 - 2bc\cos A$.
- Pythagoras' theorem: (A special case of the cosine rule) If ABC is a right-angled triangle, with the right-angle at A, then $a^2 = b^2 + c^2$.
- The two angles at the base of an isosceles triangle are equal. Conversely, if two angles in a triangle are equal then the triangle is isosceles.

Triangle Area Formulae

- $\frac{1}{2}ah_A$
- $\frac{1}{2}ab\sin C$
- $2R^2\sin A\sin B\sin C$
- $\frac{abc}{4R}$
- sr
- $\sqrt{s(s-a)(s-b)(s-c)}$

Other Results

- Opposite sides of a parallellogram are equal. Conversely, if a quadrilateral has both pairs of opposite sides equal, then it is a parallellogram.
- Opposite angles of a parallellogram are equal. Conversely if both pairs of opposite angles in a quadrilateral are equal, then the quadrilateral is a parallellogram.